*tape over page 172

Section 7.2B

Key Concept — *Definition of nth Root*

- **Words** For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.
- **Example** Since $2^5 = 32$, 2 is a fifth root of 32.

$2^5 = 32$ $\quad \sqrt[5]{32} = 2$

$9^2 = 81$ $\quad \sqrt{81} = 9$

The symbol $\sqrt[n]{}$ indicates an nth root.

$\sqrt[n]{50}$

index — radical sign — radicand

$\sqrt{} = \sqrt[2]{}$

When an expression has a power and an nth root it can be written 2 different ways, but they mean the same thing.

Radical Notation ⟶ Rational Exponent Notation

$$\sqrt[n]{x^m} = x^{\frac{m}{n}} \quad \text{OR} \quad \left(\sqrt[n]{x}\right)^m = x^{\frac{m}{n}}$$

$\sqrt[3]{10} = 10^{1/3}$ $\quad \sqrt{9} = 9^{1/2} = 3$

$9^2 = 81$

Example:

Rewrite the following expressions using rational exponent notation.

$\sqrt[6]{64} = 64^{1/6}$

$\left(\sqrt[5]{243}\right)^2$

$243^{2/5}$

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4) Write each radical as a power.

a) $\sqrt[3]{7}$

$7^{1/3}$

b) $\sqrt[5]{x}$

$x^{1/5}$

c) $\sqrt{y}$

$y^{1/2}$

5) Write each power as a radical.

a) $8^{1/4}$

$\sqrt[4]{8}$

b) $z^{1/5}$

$\sqrt[5]{z}$

c) $m^{1/8}$

$\sqrt[8]{m}$

10) Write each expression in rational exponent form. Show your work and simplify your answer, if possible.

a) $\left(\sqrt{5}\right)^4$

$5^{4/2} = 5^2 = 25$

b) $\left(\sqrt[5]{x}\right)^8$

$x^{8/5}$

11) Complete the missing cells of the table below. Be prepared to share your solutions and methods.

Expression using ***rational exponent notation***	Expression using ***radical notation***
$5^{3/4}$	$\left(\sqrt[4]{5}\right)^3$ OR $\sqrt[4]{5^3}$
$2^{3/4}$	$\left(\sqrt[4]{2}\right)^3$
$m^{8/9}$	$\left(\sqrt[9]{m}\right)^8$ OR $\sqrt[9]{m^8}$
$y^{2/3}$	$\left(\sqrt[3]{y}\right)^2$ OR $\sqrt[3]{y^2}$
$y^{10/5} = y^2$	$\left(\sqrt[5]{y}\right)^{10}$

7.2C Simplifying Radical Expressions and Expressions with Rational Exponents

*tape onto page 182

When there are variables in the radicand, we need to be careful how we simplify. Note that if we incorrectly simplify $\sqrt{(-3)^2}$, we might be tempted to say it is -3. But recall from Unit 5 that $\sqrt{(-3)^2}$ is a *positive* square root and the resulting simplification also needs to be positive. This leads to:

When you find the n[th] root of an even power and the result is an odd power you must take the absolute value to ensure that the answer is nonnegative.

$$\sqrt{(-5)^2} = |-5| \text{ or } 5 \qquad \sqrt{(-2)^6} = |(-2)^3| \text{ or } 8$$

If the result is an even power or you find the n[th] root of an odd power, there is no need to take the absolute value.

SIMPLIFY -

1. apply the properties → page 184 or PINK HANDOUT
2. remove any perfect nth powers (perfect squares, cubes, etc.)
3. use absolute value symbols when the exponent is simplifying from an even power to an odd power on a variable

ex.1 $\sqrt{b^2} = |b|$

ex.2 $\sqrt[4]{16x^{12}} = 2|x^3|$

$x^{12/4}$

*This is because a number raised to an odd power has the ability to be positive or negative, but a number raised to an even power can only be positive. If a negative number is plugged in for the variable the two expressions would not be equal without the absolute value symbols.

If we chose $x = -3$....

$\sqrt[4]{16(-3)^{12}} = 54$

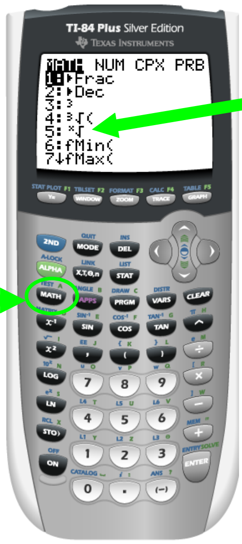
with no abs value → $2(-3)^3 = -54$

$2|(-3)^3| = 54$

match

Radicand Notation on the Calculator

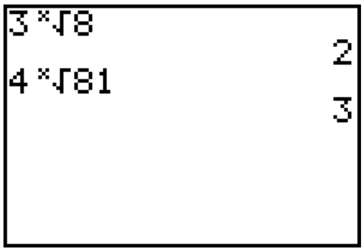
1. Type the number for your index value



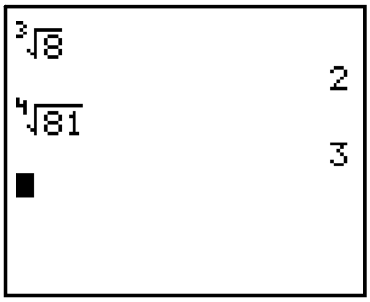
2. Press MATH

3. Choose #5

4. Type the radicand value and press ENTER



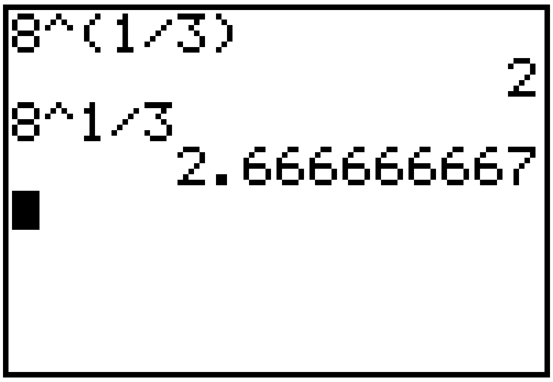
newer operating system



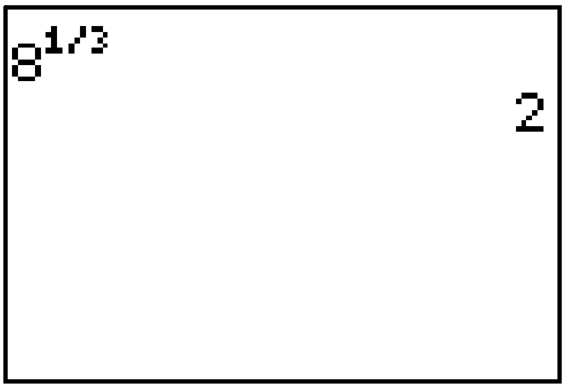
Rational Exponent Notation on the Calculator

correct!

not correct :(



newer operating system



Properties of Rational Exponents

	PROPERTY
1.	$a^m \cdot a^n = a^{m+n}$
2.	$(a^m)^n = a^{mn}$
3.	$(ab)^m = a^m b^m$
4.	$a^{-m} = \frac{1}{a^m},\ a \neq 0$
5.	$\frac{a^m}{a^n} = a^{m-n},\ a \neq 0$
6.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},\ b \neq 0$

New

Properties of Radicals

$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ **Product property**

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ **Quotient property**

Examples: page 183

Simplify. Use absolute value symbols where required.

$216^{\frac{2}{3}}$

$\left(\sqrt[3]{216}\right)^2$

$(6)^2$

$\boxed{36}$

$\sqrt[5]{32^3}$

$\left(\sqrt[5]{32}\right)^3$

$(2)^3$

$\boxed{8}$

$625^{\frac{-3}{4}}$

$\left(\sqrt[4]{625}\right)^{-3}$

$(5)^{-3}$

$\frac{1}{5^3} = \boxed{\frac{1}{125}}$

$\sqrt[3]{8x^6}$

$2x^{6/3}$

$\boxed{2x^2}$

$\sqrt[4]{2401y^4z^{12}}$

$7y^{4/4}z^{12/4}$

$\boxed{7|y'|z^3|}$

$\sqrt[2]{49a^4b^2}$

$7a^{4/2}b^{2/2}$

$\boxed{7a^2|b'|}$

$(125x^6y^3)^{\frac{-1}{3}}$

$125^{-1/3}x^{-6/3}y^{-3/3}$

$\frac{1}{125^{1/3}x^{6/3}y^{3/3}} = \boxed{\frac{1}{5x^2y'}}$

$(1024a^5)^{\frac{4}{5}}$

$\left(\frac{16r^8}{81}\right)^{\frac{3}{4}}$

$\sqrt[5]{\left(\frac{243}{x^5}\right)^2}$

$\left(729x^9y^3\right)^{\frac{2}{3}}$

$\left(\frac{81d^4}{1296}\right)^{\frac{3}{4}}$